Math 375
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Sets

Some of the material in this worksheet was taken from *A Concise Introduction to Pure Mathematics, 2 ed.* by Martin Liebeck.

One could argue that sets are the most fundamental objects in mathematics. A ***set*** is a collection of objects, which are called the ***elements*** of the set. If S is a set and x is an element of S, we denote that by $x \in S$. If x is not an element of S, then we denote that by $x \notin S$. There are several sets that you are already familiar with, e.g. the integers $\mathbb{Z}$, the rational numbers $\mathbb{Q}$, and the real numbers $\mathbb{R}$. Another class of sets that we will be working with are vector spaces. More about those later.

There are several ways of describing a set. One way is to simply list the elements of the set within curly braces. For example, $\{1, 2, 3\}$ is the set of the objects 1, 2, and 3. Another example is the set $\{1, \{2\}\}$ whose elements are the number 1 and the set $\{2\}$. That's right, sets can be elements of other sets!

Often just listing the elements of our set is not a convenient way to describe it. For example, the set of all real numbers whose square is less than 2 cannot easily be described by listing all the elements of this set. Set builder notation gives us a better way to describe sets like this one. Using set builder notation, we can describe the set of all real numbers whose square is less than 2 by

$$\{x \in \mathbb{R} \mid x^2 < 2\}.$$

The part to the left of the $|$ describes the set that our objects initially belong to, and the part to the right of the $|$ describe the condition (or conditions) that we are further imposing on these elements. Sometimes people use a colon, :, instead of a bar $|$, so alternatively our set could be written as

$$\{x \in \mathbb{R} : x^2 < 2\}.$$

A special set is the set of no elements. This set is called the ***empty set*** and is denoted by $\emptyset$.

Two sets are defined to be equal when they consist of exactly the same elements. For example,

$$\{1, 3, 5\} = \{5, 3, 1\} = \{1, 1, 5, 3, 3, 5, 1, 1, 1\}.$$

We say that a set T is a ***subset*** of a set S if every element of T is an element of S. For example, the subsets of $\{1, 2\}$ are

$$\{1, 2\}, \{1\}, \{2\}, \text{ and } \emptyset.$$

By convention, $\emptyset$ is a subset of every set.

Given two sets, there are a couple of ways that one can make a new set from these existing sets. If A and B are sets, then the ***union*** of A and B, denoted $A \cup B$, is the set of all elements that are either elements of A or elements of B. That is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

For example, if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \cup B = \{1, 2, 3\}$.

The ***intersection*** of two sets A and B, denoted $A \cap B$, is the set consisting of all elements that are elements of both A and B. That is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

For example, if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \cap B = \{2\}$.

If T is a subset of S, then the ***complement*** of T in S, denoted $S - T$ or $S \backslash T$, is the set of all elements of S that are not elements of T. That is

$$S - T = S \backslash T = \{x \in S \mid x \notin T\}.$$

For example, if $S = \{1, 2, 3\}$ and $T = \{1, 2\}$, then $S - T = \{3\}$.