Math 375
9/14/15

Vector Spaces

A set V, equipped the with the operations of addition and scalar multiplicaiton, is a vector space if it satisfies the following axioms.

1. *Closure under addition* For every pair of elements $\mathbf{x}$ and $\mathbf{y}$ in V, $\mathbf{x}+\mathbf{y} \in V$.
2. *Closure under scalar multiplication* For every $\mathbf{x} \in V$ and every real number a, $a\mathbf{x} \in V$.
3. *Commutativity of addition* For every $\mathbf{x}, \mathbf{y} \in V$, $\mathbf{x}+\mathbf{y} = \mathbf{y}+\mathbf{x}$.
4. *Associativity of addition* For all $\mathbf{x}, \mathbf{y}, \mathbf{z} \in V$, $(\mathbf{x}+\mathbf{y})+\mathbf{z} = \mathbf{x}+(\mathbf{y}+\mathbf{z})$.
5. *Existence of zero vector* There exists an element, that we denote by $\mathbf{0}$, in V such that $\mathbf{x}+\mathbf{0} = \mathbf{x}$ for all $\mathbf{x} \in V$.
6. *Existence of additive inverses* For every $\mathbf{x} \in V$, the element $(-1)\mathbf{x}$ has the property $\mathbf{x}+(-1)\mathbf{x} = \mathbf{0}$.
7. *Associativity of scalar multiplication* For every $\mathbf{x} \in V$ and real numbers a and b, we have $a(b\mathbf{x}) = (ab)\mathbf{x}$.
8. *Distributive law for vector addition* For all $\mathbf{x}, \mathbf{y} \in V$ and all real numbers a, we have $a(\mathbf{x}+\mathbf{y}) = a\mathbf{x}+a\mathbf{y}$.
9. *Distributive law for scalar addition* For all $\mathbf{x} \in V$ and all real numbers a and b, we have $(a+b)\mathbf{x} = a\mathbf{x}+b\mathbf{x}$.
10. *Existence of multiplicative identity* For every $\mathbf{x} \in V$, we have $1\mathbf{x} = \mathbf{x}$.