Math 375
Worksheet 1 - Proof writing
9/3/2015

What makes a proof 'good'?

Directions: The following statement is a very standard introduction to proof-writing homework question:

Show that $\sqrt{2}$ *is not a rational number*

Below are two different answers, all of which would fail to get full credit. Identify what is good and bad about each of them; be specific and constructive. Then, write a better proof.

Proof 1: To show that $\sqrt{2}$ is not a rational number, we only need to show that it is an irrational number. If we consider its decimal expansion, we can write $\sqrt{2}$ as a series of the form:

$$1 + \frac{a_1}{10} + \frac{a_2}{100} + \ldots + \frac{a_n}{10^n} + \ldots.$$

The values a_n are all distinct and nonzero.

Since any irrational number can be written as an infinite series of this form, $\sqrt{2}$ is irrational.

Comments:

Proof 2: If $\sqrt{2}$ was rational, then $q^2 = 2$ for some fraction q. Then, the numerator of q squared is even, but only the square of even number can be even. Then the numerator of q^2 is divisible by 4. If we divide both sides by 4, we get some new fraction q' so that $(q')^2 = 1/2$, so the reciprocal of $(q')^2$ is q^2. Since $q^2 = 4(q')^2$ and $q^2 = 1/(q')^2$, say WLOG that $q > 1$. Then the second equation says $q' < 1$, but the first equation says the opposite. Similarly, if $q' > 1$. Thus both equalities cannot be simultaneously satisfied by any pair of rational numbers and so no such q exists and $\sqrt{2}$ is not rational.

Comments:

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