

Vector Spaces

Determine whether or not the following sets are vector spaces.

1. $V = \{\mathbf{0}\}$.

2. $V = \mathbb{R}^2$ where addition is defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 y_2)$ and scalar multiplication is defined by $a(x, y) = (ax, y)$.

3. $V = \mathbb{R}^2$ where addition is defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + 2x_2, y_1 + 3y_2)$ and scalar multiplication is defined by $a(x, y) = (ax, ay)$.

4. $V = \mathbb{R}^2$ where addition is defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication is defined by

$$a(x, y) = \begin{cases} (0, 0) & \text{if } a = 0 \\ (ax, \frac{y}{a}) & \text{if } a \neq 0. \end{cases}$$