

## Linear Independence

For 1–4, determine whether the following sets are linearly dependent or linearly independent.

1.

$$\left\{ \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}, \begin{pmatrix} -2 & 6 \\ 4 & -8 \end{pmatrix} \right\} \text{ in } M_{22}(\mathbb{R})$$

2.

$$\left\{ \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 2 & -4 \end{pmatrix} \right\} \text{ in } M_{22}(\mathbb{R})$$

3.  $\{(1, -1, 2), (1, -2, 1), (1, 1, 4)\}$  in  $\mathbb{R}^3$ .

4.  $\{(1, -1, 2), (2, 0, 1), (-1, 2, -1)\}$  in  $\mathbb{R}^3$ .

5. Let  $\mathbf{u}$  and  $\mathbf{v}$  be distinct vectors in a vector space  $V$ . Prove that  $\{\mathbf{u}, \mathbf{v}\}$  is linearly dependent if and only if  $\mathbf{u}$  or  $\mathbf{v}$  is a multiple of the other.

6. Prove that a set  $S$  of vectors is linearly independent if and only if each finite subset of  $S$  is linearly independent.