Math 375
Worksheet 4 - Inner Products and Norms
9/28/15

Inner Products and Norms

1. Let V be a euclidean space.

   (a) Prove that $\langle x + y, z\rangle = \langle x, z\rangle + \langle y, z\rangle$ for all $x, y, z \in V$.

   (b) Prove that $c\langle x, y\rangle = \langle x, cy\rangle$ for all $x, y \in V$ and $c \in \mathbb{R}$.

2. Let β be a basis for a euclidean space V.

(a) Prove that if $\langle x, z \rangle = 0$ for all $z \in \beta$, then $x = 0$.

(b) Prove that if $\langle x, z \rangle = \langle y, z \rangle$ for all $z \in \beta$, then $x = y$.

3. Let V be a euclidean space, and suppose that x and y are orthogonal vectors in V. Prove that $||x+y||^2 = ||x||^2 + ||y||^2$. Can you deduce the Pythagorean theorem from this?

4. Suppose that $\langle \cdot, \cdot \rangle_1$ and $\langle \cdot, \cdot \rangle_2$ are two inner products on a vector space V. Prove that $\langle \cdot, \cdot \rangle = \langle \cdot, \cdot \rangle_1 + \langle \cdot, \cdot \rangle_2$ is another inner product on V.